MEMORANDUM

Date: November 29\textsuperscript{th}, 2006

To: Mr. Kazunari Maeda

From: Brent Taft and Jesse McAvoy

Subject: Determination of the Temperature Dependent Spring Constant of a Nitinol Expansion Spring
REPORT SUMMARY

The objective of this experiment was to determine the temperature dependent spring constant of a Nitinol expansion spring. Nitinol, Nickel Titanium Naval Ordnance Laboratory, is the best known and most widely used of the class of smart materials known as shape memory alloys (SMA). A SMA is a material that when deformed, within limits, will regain its original shape when heated. SMA undergo a solid-to-solid phase transformation and exhibit very different properties above and below their transition temperature. This solid-to-solid phase transformation from the martensite to austenite phases gives a SMA spring two very different spring constants; a SMA spring is one form of thermovariable rate spring.

The spring constant of the Nitinol expansion spring was evaluated at various temperatures in order to determine the spring constant associated with the martensite and austenite phases of this spring. The average spring constant for the low temperature, martensite phase, was determined to be 183 N/m and the average spring constant for the high temperature, austenite phase, was determined to be 220 N/m. Additionally, the transition temperature was determined to be approximately 44°C. An uncertainty analysis shows that these results were achieved with a high degree of precision. The results are consistent with the literature.
BACKGROUND*

A SMA is a material that when deformed, within limits, will regain its original shape when heated. Part of what makes the shape memory effect so powerful is that the original or base shape of the material can be engineered to almost any configuration (e.g. wires, springs, and even complex 3D geometries). The unique properties of SMA are the result of a temperature dependent solid-to-solid crystallographic phase change. This crystallographic, or martensitic, transformation occurs as the material passes through a set temperature. Above the transformation temperature the material is in the austenite phase. The atomic lattice of the austenite, or parent, phase is a body-centered cubic. The ordered cubic structure of the austenite phase helps make it the stronger of the two material phases. Below the transformation temperature the SMA is in the martensite phase; as the SMA enters this phase it undergoes a reversible atomic-scale distortion. During this distortion, the atoms of the SMA “shear,” forming a complex three-dimensional arrangement (Kauffman & Mayo, 6). The transformation from the austenite to martensite phases does not necessarily cause a visible shape change. The characteristic SMA ‘shape change’ is the metamorphosis that the alloy undergoes as it thermally recovers to the ‘remembered’ austenite base shape after undergoing a deformation and subsequent shape change in the martensite phase. It is important to mention that some SMA have been trained to undergo a two-way shape change in which they remember a shape for the austenite and martensite phases, as opposed to the one-way shape change, mentioned previously, where the SMA only remembers the austenite shape (Wikipedia “Shape Memory Alloys”).
Due to the difference in atomic structure between the two phases, the SMA exhibits very different stress-strain behaviors in its two phases. The austenite phase exhibits a ‘normal’ stress-strain curve while the martensite phase exhibits an odd stress-strain curve that plateaus between two yield points. The stress-strain plateau can be as large as 8% strain with little stress increase (Stoeckel & Waram, 383). Beyond the plateau region SMA exhibit normal plastic deformation. The austenite phase is the stronger of the two phases. The martensite phase is superelastic. This elastic plateau region can be seen in Figure 1. The martensite phase exhibits non-conventional deformation in the plateau region; this non-conventional deformation is the deformation that can be thermally recovered by heating the SMA. Shape change in the plateau region of the martensite phase is the result of the movement of individual atoms within the grains of the SMA. Whereas the shape change in non-memory metals, and beyond the plateau region in SMA, is the result of the movement of entire grains of the metal; the atoms in non-memory metal are rigidly locked into their lattice so that they cannot move (Kauffman & Mayo, 9).
The transformation temperature range is characterized by four values: the martensite start temperature \((M_s)\), the martensite finish temperature \((M_f)\), the austenite start temperature \((A_s)\), and the austenite finish temperature \((A_f)\) (Stoeckel & Waram, 383). The start and finish temperatures are the temperatures at which the transformation to each phase starts and finishes. Like all other thermal actuators, SMA exhibit some hysteresis in their phase change temperatures. The thermal hysteresis of the martensitic transformation is graphically represented in Figure 2. It is important to note that as a result of hysteresis the \(M_f\) is not the same temperature as the \(A_s\); that is, the phase change from Austenite to Martensite occurs at a slightly different temperature than the phase change from Martensite to Austenite.
The near-equiaxial nickel-titanium (Ni-Ti) alloys are the most important of the SMA. These alloys were first introduced under the trade name Nitinol (Nickel Titanium Naval Ordnance Laboratory) after being discovered and developed at the Naval Ordnance Laboratory in the late 1950’s and early 1960’s. The Ni-Ti SMA has become the preeminent SMA in large part due to its heat recovery of large strains, its biocompatibility, reasonable cost and its wide and easily adjusted range of transformation temperatures (Kauffman & Waram, 9). The transformation temperatures of Ni-Ti alloys can vary between -148°F and 212°F (-100°C and 100°C) with a hysteresis width within the range of 3.6°F and 270°F (2°C and 150°C) (Stoeckel & Waram, 383). The transformation temperature is adjusted by varying the nickel-titanium ratio or by adding a third element (e.g. Cobalt and Iron) to adjust the elemental ratio. The austenite ‘base’ shape of Nitinol can be set by heating the alloy to approximately 932°F (500°C) while constraining it to the desired configuration. This heating effectively forces the atomic lattice of the alloy to adopt the ordered atomic structure of the austenite phase while in that configuration (Kauffman & Mayo, 8). Other SMA include copper-zinc (brass), copper-zinc-aluminum, and gold-cadmium (Kauffman & Mayo, 9).

*Figure 2. Thermal hysteresis in the martensitic transformation (Stoeckel & Waram, 383)*
SMA technology has been implemented in a vast range of products in many fields. SMA have been used in eyeglass frames, fire-suppression sprinklers, greenhouse louvers, and even in biomedical products such as a pulmonary embolism filter (Kauffman & Mayo, 13).

PROCEDURES

Experiment Setup

Water heated by a hot plate is used to regulate the temperature of the spring during testing. The spring must be suspended by a stout wire and support rod. Use of a stout support system prevents the addition of error in the change of length of the spring. The spring is fully submerged in the water bath so that it may be assumed that the spring is at the same temperature as the water bath.

![Experiment Setup](image)

*Figure 3. Experiment Setup*

Test Procedure

In order to determine the temperature dependent spring constant of a Nitinol expansion spring it is important to take force vs. displacement measurements over a range
of temperatures. The spring constant is found by taking the slope of the force vs. displacement graph; that is

\[ \Delta F = k \times \Delta x \]  

(1)

where \( \Delta F \) is the change in force placed on the spring, \( \Delta x \) is the change in spring displacement, and \( k \) is the spring constant. Taking the differential applied force and spring displacement helps to avoid the introduction of error cause by any prior loading of the spring: it is quite apparent to an astute observer that the spring will only fully return to its closed, or collapsed, base shape when it is in the high temperature phase. Using the differential force and displacement values eliminates any errors associated with trying to determine a zero displacement for the spring.

The spring manufacturer, Images SI Inc., claims that the shape memory phase change of their Nitinol expansion spring is in the range of 45-55°C (Images SI, Inc.). Displacement vs. force measurements will be taken over the temperature range of 40-60°C and the temperature will be measured using a handheld temperature probe manufactured by Omega. This temperature range should allow for calculation of spring constants corresponding to the phase transition region as well as the martensite and austenite phases.

At each temperature step, two different weights need to be used to displace the spring. 100 and 200 gram weights were used to deflect the spring; however, as long as the spring is not overstretched, the mass of the weights used is not important so long as they are far enough apart to give a good force vs. displacement curve. The displacement of the spring is measured directly by placing a ruler in the water bath. Data points were collected as the temperature was raised and lowered. This heating/cooling process
allowed two sets of data points to be collected. This process also allowed the researchers
the chance to look for hysteresis in the data.

The force exerted on the spring can be calculated using equation (2).

\[ F = m \times g - V \times \gamma \]  

(2)

In equation (2) \( F \) is the force exerted on the spring, \( m \) is the mass of the weight, \( g \) is
gravity, \( V \) is the volume of the weight, and \( \gamma \) is the specific weight of water (Munson et
al., 70). The volume of the weight was calculated by placing the weight in a water filled
beaker and recording the volumetric displacement of the water. This method of
calculating the volume of the weight allowed for very precise volume measurements.
However, this level of accuracy was not needed since the buoyant force accounts for less
than 4\% of the total force placed on the spring.

**RESULTS AND DISCUSSION**

An initial analysis of the collected data made it apparent that two of the data
points should be considered unreasonable outliers. The modified three-sigma test was
employed to confirm these two points as outliers (Figliola & Beasley, 138). The modified
three-sigma test classifies a point as an outlier if it meets the conditions of equation (3).

\[ N(0.5 - P(z_0)] \leq 0.1 \]  

(3)

\( Z_0 \) is given by equation (4) where \( x_i \) is the data point, \( \bar{x} \) is sample mean, and \( S_x \) is the
sample standard deviation.

\[ Z_0 = \left| \frac{x_i - \bar{x}}{S_x} \right| \]  

(4)

The two faulty data points were removed from the data set, and rest of the data was
analyzed.
The spring constant at each of the temperature steps was determined using equation (1). The spring constant was graphed as a function of temperature; the thermovariable nature of this spring is readily apparent in Figure 4. The blue data points were taken as the spring was heated up to 56°C and the pink data points were taken as the spring was cooled back down to 37°C. The temperature and force measurements were not accurate enough to notice any hysteresis in the properties of this spring. In both the case of heating and cooling, the transition temperature is approximately 44°C. Images SI Inc. stated that the transition temperature of the spring was in the range of 45°C to 55°C; the transition temperature measurement obtained in this experiment is consistent with their data.

**Figure 4. Thermovariable Spring Constant Test Results**
Figure 4 also depicts the average spring constant at temperatures above and below the transition temperature as well as the uncertainty of the average spring constant below the transition temperature. The average spring constant for the low temperature, martensite phase, is 183 N/m. The average spring constant for the high temperature, austenite phase, is 220 N/m. These values are consistent with the theory; the austenite phase of SMA is the stronger of the two phases (see Figure 1). The standard deviation of the average spring constant above the transition temperature is essentially zero; therefore, the uncertainty of the spring constant above the transition temperature is negligible. The standard deviation of the average spring constant below the transition temperature is 15.66 N/m. This value was used in the Student-t Distribution to calculate the uncertainty of the spring constant below the transition temperature. The Student-t Distribution determines the probability that a data point will lie within a specific range; equation (5) was used to plot the 95% confidence interval (Figliola & Beasley, 121).

\[ x_i = \bar{x} \pm t_{v, p} \times S_e \]  

\( t_{v, p} \) depends on the degrees of freedom of the data set and the confidence interval of interest; this experiment contained 7 degrees of freedom and the 95% confidence interval was of most interest. The results of this experiment are very accurate and are consistent with the background theory.

**CONCLUSION**

This experiment was very successful. The thermovariable spring constant of this particular Nitinol spring was measured quite accurately. Additionally, the transition temperature of this spring was determined. However, the data collected as the spring was
heated and cooled was not accurate enough to note hysteresis in the phase change temperature of the spring.

Future testing could look at a larger sample of the Nitinol expansion springs offered by Images SI Inc. This testing could focus on determining the variation and/or consistency of the spring constant of these springs; it would be interesting to determine the standard deviation of the spring constant. The transition temperature of the SMA spring is certainly dependent on the exact composition, or batch, of the alloy; however, the thermovariable spring constant may not be as dependent on the exact composition of the SMA.
REFERENCES


*The Background section of this report is a slightly edited version of the third draft of Chapter 2.1.1 of Brent Taft’s Honors Thesis Proposal: Design of a passive, thermally actuated valve for fluid control of a biologically inspired satellite heat exchanger."